USAAAO 2024 - First Round

February 10^{th}, 2024

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1. David is walking down MIT's infinite corridor (latitude $42°21'33''$) when he suddenly sees the sun aligning with the window at the end of the corridor. Being the observational master he is, David immediately pulls out his compass and measures the Sun to be at an azimuth of $245.81°$. Forgetting to bring his jacket, he is painfully reminded as he walks outside that it has been less than 6 months since the previous winter solstice. Which of the following choices is closest to the current date? Assume the corridor is parallel to the surface of the Earth.

   (a) January 15
   (b) January 30
   (c) February 15
   (d) March 20
   (e) April 1

2. Abhay looks at the light curves for two main sequence stars A and B, which you can assume are blackbodies. A has its peak at a frequency two times as high as that of B. By looking at the depth of spectral lines, Abhay can also determine that A has higher metallicity than B. Abhay makes the following statements:

   P: A has higher absolute magnitude than B.

   Q: A is older than B.

   Which of the following is true?

   (a) P and Q are true.
   (b) P and Q are false.
   (c) P is true and Q is false.
   (d) P is false and Q is true.
   (e) We don't have sufficient information for one or more of these statements.

3. Aliens in a nearby star system (located in a random direction from Earth) are looking for nearby planets using the transit method. What is the probability that they can see the Earth transit across the Sun?

   (Assume they observe our Sun over multiple years and their instruments are sensitive enough to detect any transit that occurs.)

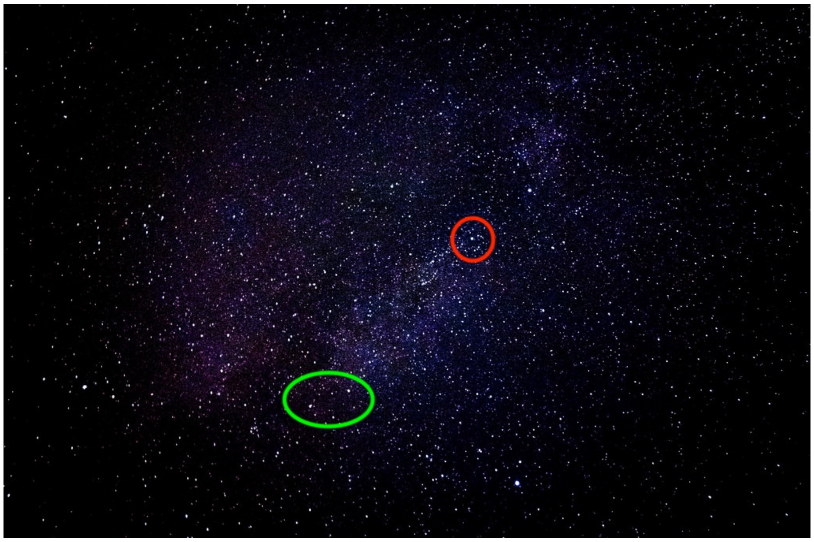
   (a) 100%
   (b) 5.8%

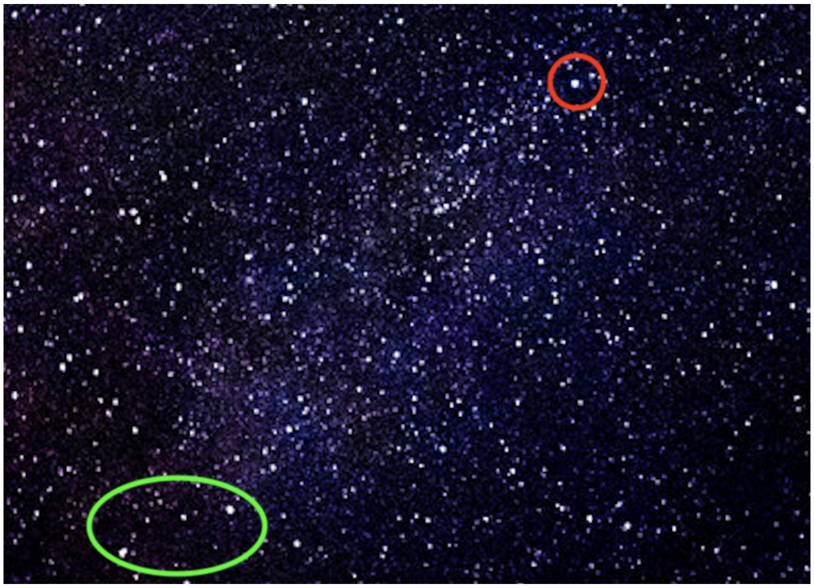
(c) 0.93%

(d) 0.47%

(e) 0.15%

4. Orion is observing the sky with two telescopes that he just made. Orion wrote down that the first telescope has a primary mirror with focal length $F_p = 2\text{m}$ and an eye piece with focal length $F_e = 30\text{mm}$. However, he does not know the specifications of his second telescope. Given that the full-field image on the left was taken by the first telescope, and the full-field image on the right was taken by the second telescope, which of the following choices could be the specifications of the second telescope?





(a) $F_p = 1\text{m}$ $F_e = 15\text{mm}$

(b) $F_p = 1\text{m}$ $F_e = 30\text{mm}$

(c) $F_p = 1\text{m}$ $F_e = 90\text{mm}$

(d) $F_p = 6\text{m}$ $F_e = 30\text{mm}$

(e) $F_p = 6\text{m}$ $F_e = 15\text{mm}$

5. A supernova is triggered largely by neutrinos. In fact, 99% of the energy coming from the supernova is released in form of neutrinos. Over a time span of about three months, the supernova outputs visible light with power equivalent to 10 billion Suns. Assuming supernova neutrinos have mean energy of around 10 MeV, that all the power of the supernova is released during the time it is visible, and that all of the power released is released in the form of either visible light or neutrinos, estimate the number of neutrinos released.

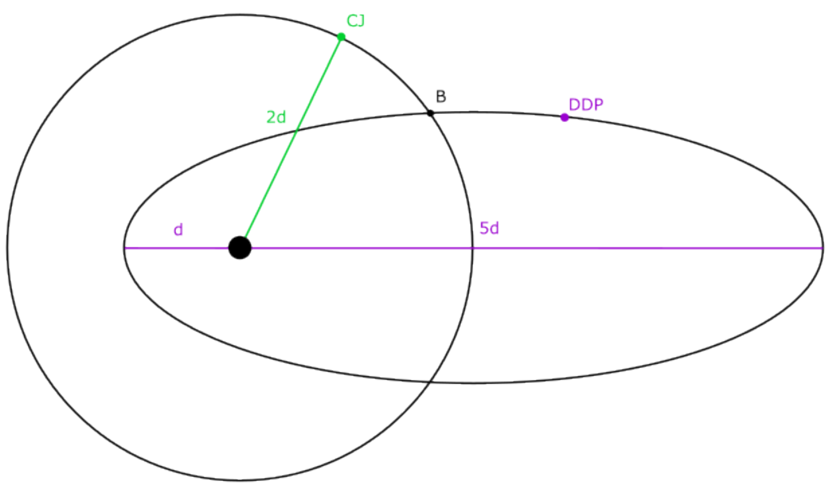
   (a) 10^{54}

   (b) 10^{55}

   (c) 10^{50}

   (d) 10^{57}

   (e) 10^{60}



6. Newly discovered planets DDP and CJ are found to orbit a nearby star, as shown in the figure above. Planet CJ has a circular orbit with a radius of $2d$, while planet DDP moves in an elliptical orbit with an aphelion of d and a perihelion of $5d$. Their orbits intersect at location B in the figure. Additionally, through external analysis, planet DDP is found to be three times more massive than planet CJ. From the perspective of the star, what is the ratio of the angular momentum of planet DDP when it passes through point B to the angular momentum of planet CJ when it passes through point B? You may assume that the masses of both planets are significantly smaller than the mass of the star they orbit.

   (a) $\frac{3\sqrt{3}}{2}$

   (b) $\sqrt{\frac{15}{2}}$

(c) $2\sqrt{3}$

(d) $\sqrt{30}$

(e) $\sqrt{\frac{10}{3}}$

7. Arjun launches a 50 kg rocket with speed 10,405 m/s from the surface of the Earth and redirects it into a stable elliptical orbit. Upon analysis, he finds the area of the orbit to 1.438×10^{15} m^2. What is the approximate distance between the periapsis of the orbit and the center of the Earth? Assume no energy was lost in the redirection of the rocket into its new orbit.

   (a) 10000 km
   (b) 5000 km
   (c) 2000 km
   (d) 39000 km
   (e) 15000 km

8. The mass density of the Milky Way galaxy determines the orbital velocity of planets, stars, and other objects orbiting around its center. Assuming a constant surface mass density σ for the Milky Way and modeling it as a perfect circular disk, identify the dependence of the circular orbital velocity $v(r)$ of a point mass at radius r from the galaxy's center.

   (a) $1/\sqrt{r}$
   (b) $1/r$
   (c) $\sqrt{r}$
   (d) r
   (e) $r^{3/2}$

9. Let's assume that on an expedition mission to Mars, we take a telescope with 0.01 arcsecond angular resolution from earth. What is the ratio of the number of the stars we can measure the parallax distance to using this telescope on Mars compared to earth? The semimajor axis of Mars is 1.524 AU.

   (a) 0.25
   (b) 0.5
   (c) 1
   (d) 4
   (e) 8

10. Ben Chen is an alien living on a system identical to earth, except his planet's obliquity is 0°. Located at latitude 42.20°, he wants to observe M52. Due to the open cluster being so dim, Ben needs perfect conditions to observe M52. Due to atmospheric effects, M52 can only be observed above an altitude of 30°. Additionally, it must be during astronomical twilight (when the Sun is more than 18° below the horizon). Of the following dates, which is the earliest after the vernal equinox that Ben can observe the cluster? The coordinates of M52 are approximately $\alpha = 0\text{h}$ and $\delta = 60^\circ$.

    (a) April 21st
    (b) June 21st

(c) August 21st

(d) October 21st

(e) December 21st

11. Joe lives at the bottom of a vertical cylindrical hole with a radius of 10 m at a depth of 10 km below the surface. He sees the Sun directly through the opening of the hole for a couple days twice a year, around November 2nd and February 9th. Which of the following is Joe's latitude?

(a) $42°22'$N

(b) $19°27'$N

(c) $4°43'$N

(d) $14°38'$S

(e) $34°36'$S

12. In the same scenario as the question above, what is the longest possible time interval that direct sunlight reaches anywhere in the bottom of the hole in a single day?

(a) 28 sec

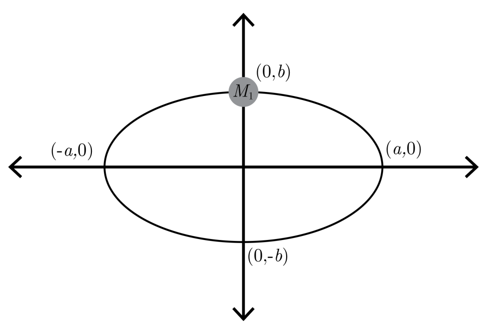
(b) 2 min 8 sec

(c) 2 min 26 sec

(d) 2 min 35 sec

(e) 2 min 41 sec

13. Samvit observes a binary star system of masses M_1 and M_2. Unfortunately, the star with mass M_2 is too dim for him to observe it, leading to the following snapshot below.



What could Samvit hypothesize to be the position and mass of the other star at this instant that would be consistent with the laws of physics and the orbit snapshot that he sees? To be clear, he has *no knowledge* of the value of M_2 or the period of the binary system.

(a) At $(\sqrt{a^2-b^2}, 0)$ with mass $M_1 + M_2$

(b) At $(0, -b)$ with mass M_1

(c) At $(\sqrt{a^2-b^2}, 0)$ with mass $\frac{M_1 M_2}{M_1+M_2}$

(d) At $(-2\sqrt{a^2-b^2}, -b)$ with mass M_1

(e) At $(\frac{M_2}{M_1}\sqrt{a^2-b^2}, 0)$ with mass M_2

14. The surface of the Sun exhibits differential rotation, with different rotational periods at different latitudes. We can measure this rotation speed using Doppler spectroscopy or by tracking the motion of sunspots.

    If the rotation speed of the Sun's surface at the equator is 2021 m/s, and at 60° South is 809 m/s, how long would it take for a sunspot at the equator to do a full extra lap around the Sun compared to a sunspot at 60° South?

    (a) 6.2 days
    (b) 25.0 days
    (c) 31.2 days
    (d) 41.7 days
    (e) 126 days

15. Two protons A and B lie in the solar interior. In the rest frame of proton A, the proton B approaches it radially from a large distance with speed $0.9c$. In the rest frame of proton A, identify the radius of the "classically forbidden" region for proton B (i.e. the region in which proton B cannot enter).

    (a) 6.6×10^{-12} m
    (b) 4.1×10^{-15} m
    (c) 2.3×10^{-15} m
    (d) 3.8×10^{-18} m
    (e) 1.2×10^{-18} m

16. Consider a sun-planet-moon system. The rotation period of the planet is 2 days. The period of revolution of the moon around the planet is 42 days while that of the planet around the sun is 420 days. What is the length of the lunar cycle as seen from the planet? You can assume the the direction of planetary rotation, planetary revolution and lunar revolution is the same.

    (a) 42.1 days
    (b) 44.3 days
    (c) 46.7 days
    (d) 50.5 days
    (e) 53.1 days

17. The Extremely Large Telescope (ELT) is an optical telescope under construction in Chile. The primary mirror has been planned to have a diameter of 39.3 m making it largest optical telescope ever built. One of the goals for this telescope is the direct imaging of exoplanets. Consider an exoplanet at a distance of 1 A.U. from a star. What is the maximum distance from Earth of such a star-exoplanet system in which the ELT can resolve the exoplanet separately from the star? Ignore atmospheric seeing and assume optical wavelength to be 500 nm.

    (a) 112 pc
    (b) 212 pc
    (c) 312 pc
    (d) 412 pc
    (e) 512 pc

18. At 6am on March 20th, as the Sun is rising, Leo, who is at $(40°\mathrm{N}, 75°\mathrm{W})$, plants a stick vertically on the ground. At that moment, he marks out a (finite) line on the ground in the direction of the shadow of the stick at that moment, labeling it with the current time. Every hour afterwards, on the hour, he marks out a new line in the current direction of the shadow, until the sun sets at 6pm.

    Three months later, Leo returns to the same spot, where the vertical stick and lines remain. Again, every hour on the hour, he marks out a line in the current direction of the shadow, until the sun sets.

    Let α_{12} and α_6 be the azimuths of the lines drawn in the spring at 12pm and 6pm, and β_{12} and β_6 be the azimuths of the lines drawn in the summer at 12pm and 6pm. Which of the following statements is true? Ignore atmospheric effects and the equation of time.

    (a) $\alpha_{12} = \beta_{12}, \alpha_6 = \beta_6$
    (b) $\alpha_{12} > \beta_{12}, \alpha_6 = \beta_6$
    (c) $\alpha_{12} < \beta_{12}, \alpha_6 = \beta_6$
    (d) $\alpha_{12} = \beta_{12}, \alpha_6 > \beta_6$
    (e) $\alpha_{12} = \beta_{12}, \alpha_6 < \beta_6$

19. Leo then realizes that, in order for a single set of hour markings to accurately describe the time over the course of an entire year, the stick may need to be tilted away from the vertical position. More specifically, consider straight lines drawn on the ground from the base of the stick; the shadow at a certain fixed time of day, on different days of the year, should always lie on the same line. Measured as an angle from the vertical, how much does the stick need to be tilted, and in which direction?

    (a) $0°$ (no tilt needed)
    (b) $40°$ towards the North
    (c) $50°$ towards the North
    (d) $40°$ towards the South
    (e) $50°$ towards the South

20. Imagine a very long cylindrical planet that has a satellite orbiting around it. Considering that the average density of the planet is ρ and the radius is R, find the expression that relates the period P of the satellite with its distance d to the center of the planet.

    (a) $\frac{2d}{R}\sqrt{\frac{2\pi}{G\rho}}$
    (b) $\frac{d}{R}\sqrt{\frac{2\pi}{G\rho}}$
    (c) $\frac{d}{2R}\sqrt{\frac{4\pi}{G\rho}}$
    (d) $\frac{d}{R}\sqrt{\frac{4\pi}{G\rho}}$
    (e) $\frac{d}{R}\sqrt{\frac{\pi}{G\rho}}$

21. The problem of magnetic monopoles — that is, the apparent absence of magnetic monopoles in the universe — arises from the fact that some modern physical theories (such as string theory) predict that the number density of magnetic monopoles at the time of their creation was $n_M(t_{GUT}) \approx 10^{82}\,\mathrm{m}^{-3}$. The inflation theory provides a possible solution to this problem, as the exponential expansion of the primordial universe would "dilute" the monopoles. Calculate, approximately, how much the universe expanded during the inflationary period so that today the probability of a single magnetic monopole existing in the observational universe is 1%. Consider that the beginning of inflation coincides with the time of the creation of magnetic monopoles, and that the universe is flat (Euclidean geometry can be used on large scales). You can use that the diameter of the observational universe is 28.5 Gpc, and that between the end of inflation and today, the universe has linearly expanded by a factor of 5×10^{26}.

(a) e^{40}
(b) e^{50}
(c) e^{55}
(d) e^{65}
(e) e^{85}

22. Tara is investigating a new interesting type of stars she decides to call the X stars. She observes that in an X star of mass M and radius R, the gas pressure in the center of the star is proportional to $\frac{M^3}{R^5}$. What is the temperature at the center of the star proportional to?

(a) MR
(b) $\frac{M}{R}$
(c) M^2R^2
(d) $\frac{M^2}{R^2}$
(e) $const$

23. VOIDED A recently discovered star *Recentus* in a nearby galaxy has been confirmed to have a luminosity of $4L_\odot$ with a radius of $4R_\odot$. What is the approximate frequency of peak emission of *Recentus*?

(a) 2×10^{14} Hz
(b) 4×10^{14} Hz
(c) 6×10^{14} Hz
(d) 8×10^{14} Hz
(e) 10×10^{14} Hz

24. Tara has become obsessed with X stars and decides to look for them in other galaxies. She observes one on the edge of a galaxy with radius $r = 15 \times 10^3 pc$. She estimates that the star is moving with at a speed of $v = 270 km/s$. What is a good estimate for the mass of the galaxy in unit mass of the sun?

(a) 1×10^6
(b) 1×10^{11}
(c) 1×10^{18}
(d) 1×10^{28}

(e) 1×10^{41}

25. There is a galaxy at redshift 0.5 for which we have a measurement for apparent bolometric magnitude to be 22. With a standard candle in that galaxy, we have found its luminosity distance to Earth to be 2.8 Gpc. Estimate the luminosity of this galaxy.

(a) $10^{10} L_\odot$
(b) $10^{12} L_\odot$
(c) $10^{11} L_\odot$
(d) $10^{13} L_\odot$
(e) $10^{15} L_\odot$

26. An astronomer wants to design a telescope so that the full moon fills the entire FOV of the telescope. She uses an eyepiece with a FOV of 60°. If the focal length of the eyepiece is 25mm, what will the focal length of the chosen telescope be?

Note that the angular diameter of the moon is 0.5°. Never look at the moon through a telescope without proper precautions!

(a) 2000mm
(b) 1500mm
(c) 6000mm
(d) 3000mm
(e) 1000mm

27. The Cosmic Microwave Background is made of light that was replied when the Universe first became transparent. It is a blackbody spectrum with temperature equal to the current temperature of the Universe. We observe the peak wavelength of the CMB to be at 1.063 millimeters. When the CMB was released, we can theoretically predict the temperature of the universe to be 3000 Kelvins. How much larger was the density of matter when the CMB was released than now? Select the closest answer.

(a) 10^3
(b) 10^6
(c) 10^9
(d) 10^{12}
(e) 10^{15}

28. **Questions 28-30 build upon the same prompt. Use data from any of the questions for any of the other questions.**
Moving into MIT for the start of the spring semester, Austin is flying from Lubbock, Texas $(33.58^\circ\text{N}, 101.84^\circ\text{W})$ to Boston, MA $(42.36^\circ\text{N}, 71.06^\circ\text{W})$. However, when he lands he finds that he is not in Boston. The pilots entered the latitude coordinate incorrectly! But Austin remembers that the plane left Lubbock at a bearing of 63°. Assume that the flight still took the shortest path to the current destination. Where is Austin now?

(a) $(51.46^\circ\text{N}, 71.06^\circ\text{W})$
(b) $(48.46^\circ\text{N}, 71.06^\circ\text{W})$
(c) $(45.46^\circ\text{N}, 71.06^\circ\text{W})$

(d) $(41.46°N, 71.06°W)$

(e) $(39.46°N, 71.06°W)$

29. After that slight headache, Austin is back at MIT in Boston! For his astronomy research, he is observing the LARES satellite which is a ball of diameter 36.4 cm made out of THA-18N (a tungsten alloy). It orbits at a distance 1450 km from the surface of the Earth and at an inclination of 69.49° relative to the equatorial plane. What is the highest altitude Austin can point his telescope if he wants to observe LARES at its highest latitude?

    (a) 9.4°
    (b) 14.4°
    (c) 18.4°
    (d) 23.4°
    (e) 33.4°

30. VOIDED Austin is now observing LARES, but in this problem he is allowed to observe LARES at any declination. Assume that LARES is a uniform spherical ball, and for the purposes of this problem, assume that LARES is a perfect blackbody; i.e. THA-18N has an albedo of 0. What is the brightest apparent magnitude Austin can observe LARES? You can use the fact that the Sun's apparent magnitude is -26.74.

    (a) 6.78
    (b) 7.77
    (c) 8.76
    (d) 9.75
    (e) 10.74