USAAAO 2023 - First Round

February 11^{th}, 2023

1. According to the astronomical Julian day count, JD 2459946.0 corresponds to January 1, 2023 at 12:00 UT. What would be the date and time at Chorzow, Poland corresponding to JD 2460000.0?

   Poland uses the Central European Time (GMT + 1:00).

   (a) 31 January 2023, 24:00
   (b) 24 February 2023, 13:00
   (c) 21 March 2023, 14:00
   (d) 21 June 2023, 18:00
   (e) 31 December 2023, 24:00

2. A cluster has a radius of 2 parsecs. A Sun-like star in the cluster has an apparent magnitude of 10. When we look at the cluster with a telescope that has an en eyepiece with a field of view of 30 degrees, the cluster just fits within the eyepiece. If this telescope has an eyepiece of focal length 20 mm, what is the focal length of its objective lens?

   (a) 570 mm
   (b) 283 mm
   (c) 144 mm
   (d) 425 mm
   (e) 301 mm

3. The radius of the Moon is about 4 times smaller than the radius of the Earth. The mean albedos of the Moon and the Earth are 0.12 and 0.36, respectively. The Mars Reconnaissance Orbiter took a picture of the Earth-Moon system. How many times brighter than the Moon did the Earth appear in the image?

   (a) 4/9
   (b) 4/3
   (c) 16/3
   (d) 12
   (e) 48

4. The Sun has a surface temperature of about 6000 K and its blackbody emission peaks in the visible spectrum. Around 1940s, astronomers found out that the Sun is a strong X-ray emitter. Today, it is understood that this emission comes from the solar corona where temperatures can reach on the order of 10^6 K. Assuming the corona is a blackbody emitter, what wavelength would that emission peak at?

   (a) 2.9 nm

(b) 2.9 μm
(c) 2.9 mm
(d) 2.9 m
(e) 2.9 km

5. Friedrich Bessel was the first person to quantitatively measure the annual change in stellar positions due to the motion of the Earth around the Sun. This change is known as stellar parallax. Bessel determined the stellar parallax for 61 Cygni to be about 1/3 arc-seconds. What is the distance to 61 Cygni from Earth?

   (a) 1/3 light years
   (b) 3 light years
   (c) 1/3 parsec
   (d) 3 parsecs
   (e) 3 kiloparsecs

6. Compute the black body luminosity of a neutron star with its surface temperature at 10^6 K and radius 10^4 m.

   (a) 0.1 $L_\odot$
   (b) 0.2 $L_\odot$
   (c) 0.3 $L_\odot$
   (d) 0.4 $L_\odot$
   (e) 0.5 $L_\odot$

7. **Fill in the blanks.**

   The Balmer lines are a series of emission spectrum of hydrogen. In the low temperature interstellar medium, despite hydrogen being the most commonly occurring element in such environment, the Balmer series is not observed as absorption lines. That happens because the transitions that require such spectrum to appear as absorption lines correspond to transitions from the _____(1)_____ hydrogen level to higher states. In low temperature environments, the _____(1)_____ level is _____(2)_____.

   (a) (1) $n = 1$, (2) always occupied
   (b) (1) $n = 1$, (2) never occupied
   (c) (1) $n = 2$, (2) always occupied
   (d) (1) $n = 2$, (2) never occupied
   (e) (1) $n = 3$, (2) never occupied

8. A comet passes near the Sun on a parabolic orbit. While it's passing near the Sun with orbital velocity V, the Sun's heat causes the comet to melt, and it shatters into many small fragments. The fragments move away uniformly in all directions (in the comet's reference frame) with velocity $v \ll V$. What fraction of the fragments will escape the solar system? Ignore any forces other than the Sun's gravity.

   (a) 0%
   (b) 50%

(c) 100%

(d) $\frac{v}{V}$

(e) $1 - \frac{v}{V}$

9. Consider Galaxies A and B, both of which have radius R. At a distance R from its center, Galaxy A's rotational velocity is equal to v. Meanwhile, Galaxy B's radial velocity dispersion is also equal to v. However, galaxy A is spiral while galaxy B is spherical elliptical and composed of uniform, evenly-spaced stars. Calculate the masses of both galaxies. (Answer choices are listed as m_A; m_B).

   (a) v^2R/G; v^2R/G

   (b) v^2R/G; $5/6v^2R/G$

   (c) v^2R/G; $5/4v^2R/G$

   (d) v^2R/G; $5v^2R/G$

   (e) $5/2v^2R/G$; v^2R/G

10. Consider a satellite that has a circular orbit with a radius of 6.0×10^8 m around Venus. Due to a failure in its ignition system, the satellite's orbital velocity was suddenly decreased to zero during a maneuver. How long does the satellite take to hit the surface of the planet? Consider that the mass of Venus is 4.67×10^{24} kg and neglect any gravitational effects on the satellite other than that from Venus.

    (a) 15 hours.

    (b) 3 days.

    (c) 11 days.

    (d) 25 days.

    (e) 37 days.

11. Consider a main-sequence star with mass $M = 9.1 \times 10^{29}$ kg, which is sustained through the proton-proton chain reaction, which operates with $\epsilon = 0.7\%$ efficiency. The hydrogen and helium fractions of this star are $f_{\mathrm{H}} = 0.71$ and $f_{\mathrm{He}} = 0.22$ at the beginning of its lifetime. Assume this star has solar luminosity and that all hydrogen can be used for fusion. Calculate the lifetime of this star.

    (a) 3.3×10^{17} s

    (b) 1.1×10^{18} s

    (c) 1.5×10^{18} s

    (d) 1.5×10^{20} s

    (e) 2.2×10^{22} s

12. A planet is in an elliptical orbit around a star. Let $r_{\min}$ be the minimum distance between the planet and star, and let $r_{\max}$ be the maximum distance between the planet and star. Suppose that $r_{\max} = 4r_{\min}$. During what percentage of the time period of each orbit is the planet at least $\frac{5}{2}r_{\min}$ away from the star?

    (a) 23%

    (b) 50%

    (c) 57%

(d) 69%

(e) 77%

13. The apparent magnitude of a star of radius $0.41R_\odot$ as observed from Earth appears to fluctuate by 0.037. That is, the difference between the maximum and minimum apparent magnitudes is 0.037. This fluctuation is caused by an exoplanet that orbits the star. Determine the radius of the exoplanet.

(a) $0.075R_\odot$

(b) $0.079R_\odot$

(c) $0.085R_\odot$

(d) $0.098R_\odot$

(e) $0.12R_\odot$

14. An empirically determined approximate formula for the lifetime of a star is given by:

$$T = \left(\frac{M_\odot}{M}\right)^{2.5} \cdot 10^{10} \text{ years}$$

where T is the stellar lifetime and M is the mass of the star.

If the very first stars in the universe formed approximately 400 million years after the Big Bang, what is the most massive such star that could still exist today?

(a) $3.6M_\odot$

(b) $2.0M_\odot$

(c) $1.3M_\odot$

(d) $0.89M_\odot$

(e) $0.75M_\odot$

15. With the technology currently available, it would take hundreds of millennia to send a human-made object to other stars. A possible solution to this problem is to use relativistic light sails, which consist of very small probes propelled by radiation pressure. It is estimated that on the reference frame of an Earth observer, these sails would take 20.0 years to reach Alpha Centauri, which is 4.37 light-years away from the Solar System. The velocity of a light sail can be assumed to be constant throughout the entire trip. How long would this trip be on the reference frame of the light sail?

(a) 18.5 years

(b) 19.0 years

(c) 19.5 years

(d) 20.0 years

(e) 20.5 years

16. Which of the following constellations is not on the sky map below?

(a) Virgo
(b) Crux
(c) Lupus
(d) Libra
(e) Corona Borealis

17. The fictional towns of Baia and Caia are located at $(66.56°N, 67.55°E)$ and $(\delta, 18.95°E)$, respectively. It is known that the spherical triangle with vertices at Baia, Caia, and the North Pole covers 6.75% of Earth's surface. Compute δ.

(a) 66.56° N
(b) 55.25° N
(c) 23.44° N
(d) 55.25° S
(e) 66.56° S

18. Now suppose that Lucas is standing still in Baia (from the previous question), and Justin is standing still on the equator. Let P_{L1} and P_{J1} be the paths of Lucas's and Justin's shadows on the summer solstice, respectively. Let P_{L2} and P_{J2} be the paths of Lucas's and Justin's shadows on the vernal equinox, respectively. Assume that the heights of Lucas and Justin are small compared to the radius of the Earth, there is no atmospheric refraction, and that the Sun is a point. Given Earth's obliquity $\varepsilon = 23.44°$, which of the following is the most specific accurate description of the shapes of each path?

(a) P_{L1} : Parabola, P_{J1} : Hyperbola, P_{L2} : Line, P_{J2} : Line
(b) P_{L1} : Parabola, P_{J1} : Hyperbola, P_{L2} : Hyperbola, P_{J2} : Line
(c) P_{L1} : Parabola, P_{J1} : Parabola, P_{L2} : Line, P_{J2} : Line
(d) P_{L1} : Hyperbola, P_{J1} : Parabola, P_{L2} : Hyperbola, P_{J2} : Line
(e) P_{L1} : Hyperbola, P_{J1} : Parabola, P_{L2} : Line, P_{J2} : Line

19. In 1995, researchers at the University of Geneva discovered an exoplanet in the main-sequence star 51 Pegasi. This was the first-ever discovery of an exoplanet orbiting a Sun-like star! When they observed the star, a periodic Doppler shifting of its stellar spectrum indicated that its radial velocity was varying sinusoidally; this wobbling could be explained if the star was being pulled in a circle by the gravity of an exoplanet. The radial velocity sinusoid of 51 Pegasi was measured to have a semi-amplitude of 56 m/s with a period of 4.2 days, and the mass of the star is known to be $1.1M_\odot$. Assuming that the researchers at Geneva viewed the planet's orbit edge-on and that the orbit was circular, what is the mass of the exoplanet in Jupiter masses?

    (a) $0.81M_{♃}$
    (b) $0.75M_{♃}$
    (c) $0.69M_{♃}$
    (d) $0.47M_{♃}$
    (e) $0.33M_{♃}$

20. Suppose that an astronomer detects an electromagnetic wave of frequency ν. Some time later, another wave with the same amplitude is received, but now with a frequency 2ν. In order to calibrate the apparatus and do the necessary calculations, the astronomer decided to calculate the intensity of the second signal relative to the first. Considering that they both came from the same place, what value did the astronomer obtain?

    (a) 1/4
    (b) 1/2
    (c) 1
    (d) 2
    (e) 4

21. Consider the binary system Kepler-16, which has the primary star Kepler-16A and the secondary star Kepler-16B. It has an orbital period $P = 41.08$ days and the measured parallax is $p = 13.29$ mas. Calculate the total mass of the stars, using the fact that their maximum angular separation measured from Earth is $\theta = 2.98$ mas and they are on an edge-on orbit.

    (a) 0.756 $M_\odot$
    (b) 0.803 $M_\odot$
    (c) 0.891 $M_\odot$
    (d) 0.987 $M_\odot$
    (e) 1.326 $M_\odot$

22. The habitable zone of a star is defined as the one where water in the liquid state can exist in the surface of a planet. Therefore, considering that the planets are ideal black bodies with fast rotation, determine the maximum eccentricity that the orbit of a planet can have so that it can be home to life. Ignore any thermodynamic effects that might happen in the atmosphere or the sidereal space. Consider that the melting point of water is 273 K and the boiling point is 373 K.

    (a) 0.274
    (b) 0.302
    (c) 0.316
    (d) 0.328

(e) 0.540

23. Billions of years from now, as the Moon moves farther away from the Earth, the Earth's axial tilt may become unstable. Imagine the Earth's tilt is such that the angle between the celestial equator and the ecliptic is 60°, rather than the current 23.44° - so the Arctic Circle is now as far south as 30° North. For an observer at 40° North, how many days out of the year would the Sun never set (also known as the "polar day")? (Ignore atmospheric refraction, and assume the Earth's orbit is circular and nothing else has changed from today.)

    (a) 28
    (b) 56
    (c) 61
    (d) 67
    (e) 113

24. Erez is designing a Newtonian telescope! The equation of the primary mirror is $y = x^2/36\text{m} - 1\text{m}$, and the telescope tube intersects the mirror at $y = 0$. What is the f-number (focal ratio) of the telescope?

    (a) f/0.75
    (b) f/1.00
    (c) f/1.25
    (d) f/1.33
    (e) f/1.75

25. Consider a hypothetical planet orbiting the Sun with an obliquity angle i (angle between the axis of rotation and the normal to the orbital plane). Assume that a year is much longer than a day for this planet.

    Define a tropical region in the planet as one where the Sun reaches the zenith at some time in its revolution period. Define a frigid region in the planet as one where there is a day when the Sun never rises.

    What is the minimum value of i for which there is a location on the planet which is both tropical and frigid?

    (a) 0°
    (b) 30°
    (c) 45°
    (d) 60°
    (e) 90°

26. A comet is approaching our solar system from the depths of space with a velocity of 10000 m/s, and if it continues moving in a straight line on its current trajectory, it will just barely graze the surface of the Sun! What is the eccentricity of the comet's orbit?

    (a) 1.00014
    (b) 1.000014
    (c) 1.0000014

(d) 1.00000014
(e) 1.000000014

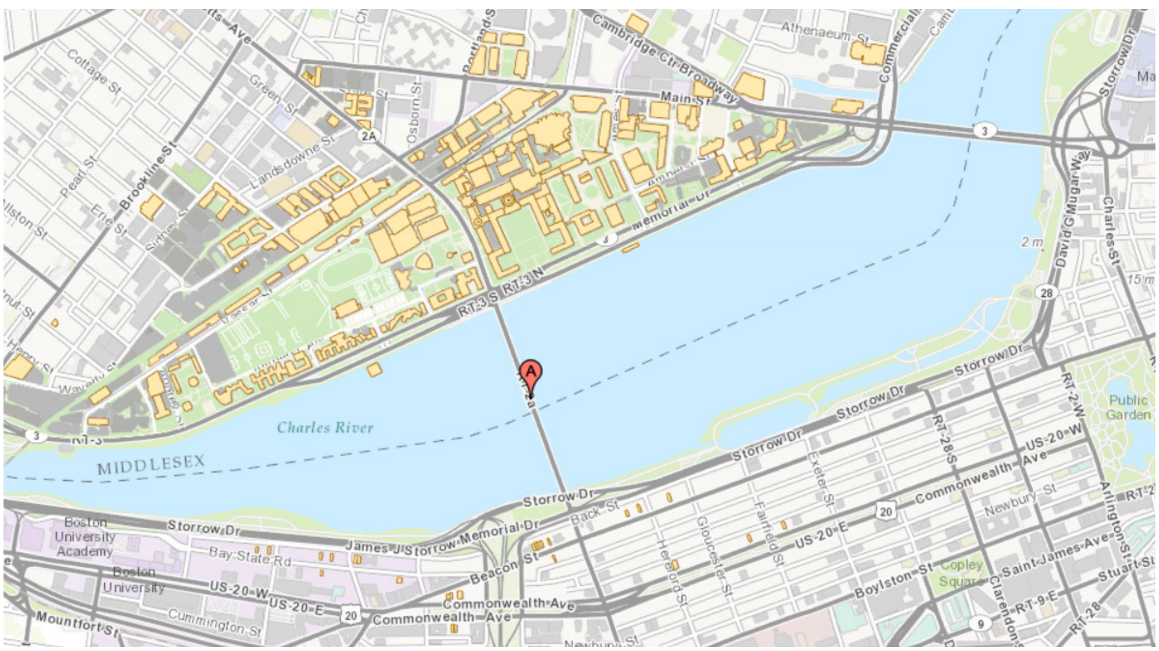
27. How far from the Solar System would a galaxy with a redshift of 0.035 be?

(a) 150 Mpc
(b) 200 Mpc
(c) 250 Mpc
(d) 300 Mpc
(e) 350 Mpc

28. Two planets A and B orbit a star with coplanar orbital paths that don't intersect. The major axes of the orbits are perfectly aligned, but the major axis of A is larger than that of B. A and B are observed to have eccentricities 0.5 and 0.75, respectively. What is the minimal possible ratio of semi-major axes of A to B?

(a) 1
(b) $\frac{7}{6}$
(c) $\frac{8}{3}$
(d) $\frac{7}{2}$
(e) 6

29. Here is a map of MIT and the surrounding area, where North points directly upwards, as taken from https://whereis.mit.edu:



Leo is biking along the Harvard Bridge (marked as "A") when he stops and looks out at the river. Looking out downriver (to the right on this map) and parallel to the banks, he sees the Sun straight in front of him, peeking out from above the buildings, and has to avert his eyes to not be blinded. What part of the academic year is it?

(a) Early fall semester (late September-early October)
(b) Late fall semester (late November-early December)
(c) Independent Activities Period (January)
(d) Early spring semester (late February-early March)
(e) Late spring semester (late April-early May)

30. After a day spent showing a visiting friend around Boston, Leo is walking back along the bridge (see diagram in previous problem) to return to Next House. The time is such that the Sun now aligns with perfectly upriver, so it is in the opposite direction compared to the morning. How high in the sky is the Sun relative to the morning?

    (a) Above the horizon, and higher than in the morning
    (b) Above the horizon, at the same altitude as in the morning
    (c) Above the horizon, but lower than in the morning
    (d) On the horizon
    (e) Below the horizon