Student's name

Problem 1. The spectrum of a blackbody peaks at a wavelength inversely proportional to its temperature. This is known as Wein's law and is used to estimate stellar temperatures. The sun can be approximated as a blackbody with its peak wavelength in the visible portion of the spectrum and a surface temperature of 6000K. Given this data, estimate the peak wavelength of a human being, assuming it to be a black body.

Solution: **B: 10 μm**

Assuming the peak wavelength for the sun to be in the middle of the visible spectrum (550 nm) and human body temperature to be about 300K, we get the required answer to be of the order of 10 μm i.e. in the infrared:

$$500\,nm \cdot 6000\,K = x\,nm \cdot 300\,K \longrightarrow x = 10000\,nm = 10\,\mu m$$

□

Doppler Spectroscopy is a technique used to detect exoplanets. The presence of a large planet causes the star to have a finite velocity around the common center of mass which leads to periodic Doppler shifts in star's spectral lines. The next two questions will pertain to this:

Problem 2. Calculate the speed of the sun around the center of mass due to the presence of Jupiter.

Solution: **B: 12 m/s**

Use the masses of sun and Jupiter to locate the common center of mass. If we set the sun at the origin, then

$$M_{\odot} \cdot 0 + m_J \cdot r_J = (M_{\odot} + m_J) r_{cm} \longrightarrow$$

Given the revolution period of Jupiter, we can get the angular velocity around the common center of mass. Using these two values, we get the speed of sun as around 12 m/s.

□

Problem 3. H_α is a prominent absorption line in the sun's spectrum with $\lambda = 656.281$. For an observer on α-Centauri, assuming the Jupiter-Sun orbital plane is in the line of sight, calculate the magnitude of the periodic shift in wavelength (nm) for the H_α line.

Solution: **A:** 2.7×10^{-5} **nm**

Use

$$\frac{\Delta_\lambda}{\lambda} \approx \frac{v}{c}$$

□

Problem 4. Why does helium burn much faster than hydrogen in a star?

Solution: **D: The energy released for each helium burning reaction is much smaller than for hydrogen**

Helium burning reaction releases much less energy per reaction than hydrogen burning reaction

□

Problem 5. The Earth's rotation axis happens to be pointing almost exactly at Polaris now, but Polaris will not always be the North Star. The direction of the rotation axis precesses with a period of 26000 years. Sometime in the future, star A, which has an angular separation of $26°11'$ from Polaris, will be the North Star. How many years from now star A will be the North star?

Solution: **D: 5000 years**

Considering a spherical triangle with vertices North Ecliptic pole (K), Polaris (P) and Star A (A), we have $KP = KA = 23.5°$, and $PA = 26°11'$ is given. Using the spherical law of cosines, $\angle AKP = 69°13'$. The Earth's rotation axis rotates around North pole with the period of 26000 years, so Star A will be the North star after $\frac{69°13'}{360°} \times 26000 \sim 5000$ years.

□

Problem 6. A 13 kg telescope is mounted on a tripod such that the angle between tripod legs is 30°. How much is the force exerted on each leg? Gravitational acceleration (g) is 9.8 m/s^2.

Solution: **B: 44.5**

If we take θ as an angle between the gravitational force and one of the legs, then the force exerted on each leg will be $\mathrm{F} = \frac{\mathrm{mg}}{3\cos\theta}$. If we project legs and vector of gravitational force (G) on a spherical sphere then θ can be derived using a spherical triangle with vertices leg1(A), leg2(B) and leg3(C). We have $AB = AC = BC = 30°$. Drawing a vertical line from G to the side of AC and naming it H, we have $\angle GHA = 90°$, $\angle AGH = 60°$ and $AH = 15°$. Using the spherical law of sines, $\theta = 17.39°$, so F = 44.5N.

□

Problem 7. Capella, the brightest star in Auriga, has celestial coordinates $05^h18^m12.78^s$, $+46°00'59.8''$. At midnight, local solar time, of the vernal equinox, which of the following is closest to the altitude of Capella above/below the horizon, as viewed from Boston? The coordinates of Boston are 42.3601° N, 71.0589° W.

Solution: **D:** +23°

Drawing the celestial sphere at midnight of the vernal equinox, let N be the celestial north pole, Z be the zenith, and C be Capella. $NZ = 90 - \phi = 47.6°$; $NC = 90 - \delta = 44.0°$; $\angle CNZ = 180 - \alpha = 100.4°$. Using the spherical law of cosines, $ZC = 66.9°$, so $h = 90 - ZC \approx 23°$.

□

Problem 8. Kerbyn is a small rocky planet in a circular orbit around a $0.2M$ star with a semimajor axis of $0.1AU$. Kerbyn has an axial tilt of $\epsilon = 42°$ and a sidereal rotation period of $05^h59^m9.4^s$. On the vernal equinox, what is the length of the apparent solar day on Kerbyn? The apparent solar day is defined as the interval between successive crossings of the meridian by the sun.

Solution: **D:** $06^h01^m45.1^s$

Kerbyn's orbital velocity is $v = \sqrt{\frac{GM}{R}}$ m/s. Let t be the difference in length between an apparent solar day and a sidereal day on the vernal equinox. Suppose we start a day with the sun at the meridian; $t_{sidereal} + t$ seconds later, the sun returns to the meridian. During this time, Kerbyn moves approximately $v \cdot (t_{sidereal} + t)$ m, so position of the celestial coordinates of the sun change by $\frac{v}{R} \cdot (t_{sidereal} + t)$ rad. Since the movement of the sun on the ecliptic is inclined at 42° to the celestial equator on the vernal equinox, the RA of the sun changes by $\frac{v \cos \epsilon}{R} \cdot (t_{sidereal} + t)$ rad. Kerbyn rotates through 2π rad in $t_{sidereal}$ s, so $\frac{2\pi}{t_{sidereal}} \cdot t = \frac{v \cos \epsilon}{R} \cdot (t_{sidereal} + t)$, or $t = 155.7$ s.

□

Problem 9. Which of the following is responsible for the opacity of the Sun's photosphere?

Solution: **D: H$^-$**

H$^-$ is the dominant source of bound free opacity in sun like or cooler stars

□

Problem 10. Planet Nine is a hypothetical planet in the outer Solar System, with a semi-major axis between 400 and 800 AU. Which of the following is a possible orbital period for Planet Nine?

Solution: **D: 15,000 years**

$T^2 = R^3$ where T is the period in years and R is the distance from the Sun in A.U. (This is true for the Sun, with mass of 1 solar mass. For a different star, the = becomes a "proportional to"). Thus, you should calculate $T^{2/3} = R$ and determine which R lies in between 400 and 800.

□

Problem 11. The main reason why Io is volcanically active is due to:

Solution: **B: Tidal forces from Jupiter**

Jupiter has one of the strongest magnetic fields in the Solar System. Io, as the closest moon to the planet, is the one that feels more effects of this (tidal forces), causing volcano activity on this moon.

□

Problem 12. Knowing that the distance between the Sun and Uranus is 2.87×10^9 km and Uranus' revolution period is 17h 14 min, determine the approximate amount of time that the Sun is above the horizon for an observer on the Uranus in the following situations:

I. At the South pole

II. At latitude 30°5'N when the declination of the Sun is 10°N

Solution: **C: I = 42 years and II = 9h 28 min**

Using the 3rd law of Kepler we find that the period of Uranus orbit is approximately 84 years. Knowing that Uranus is tilted almost 90, we know that the Sun is above the South Pole's horizon of this planet for almost half of the orbital period, so the answer for part I is 42 years. II. Using spherical trigonometry to find the time it takes to the Sun leave the horizon and and cross the meridian we have:

$$\cos(90°) = \sin(\Phi)\sin(\delta) + \cos(\Phi)\cos(\delta)\cos(H)$$
$$\cos(H) = -\tan(\Phi)\tan(\delta)$$
$$\cos(H) = -\tan(30°5')\tan(10°)$$
$$\cos(H) = -0.102$$
$$H = 98.86°$$
$$\frac{98.89°}{360°} = \frac{time}{17h14min} \rightarrow time = 4.73h$$

The total time the Sun is above the horizon will be 24.73 = 9.46h = 9h28min.

□

Problem 13. The angular diameter of star A with apparent bolometric magnitude of 2 is 2.5 times greater than the angular diameter of Star B with apparent bolometric magnitude of 7. What is the ratio of the temperature of star A to that of star B?

Solution: **C: 2**

The flux of a star can be obtained by $f = \frac{4\pi R^2 \sigma T^4}{4\pi d^2} = (\frac{R}{d})^2 \sigma T^4 = \alpha^2 \sigma T^4$, where α is the angular radius of the star. We also know that $m_2 - m_1 = 2.5\log\frac{f_1}{f_2}$, so $m_2 - m_1 = 2.5\log[(\frac{\alpha_1}{\alpha_2})^2(\frac{T_1}{T_2})^4]$. Therefore the ratio of the temperatures will be $\frac{T_1}{T_2} = 2$.

□

Problem 14. As a consequence of the virial theorem, how does the stellar temperature (T) change if we add more arbitrary energy (E) to the star?

Solution: **B: Decreases**

If we add E amount of more energy, the stellar thermal energy decreases by E. Therefore the temperature decreases.

□

Problem 15. How do we increase the reaction time for hydrogen fusion at the stellar core leaving other factors unchanged?

Solution: **C: Increase the amount of hydrogen atoms**

□

Problem 16. This problem was canceled.

Problem 17. This problem was canceled.

Recently, the exoplanet Proxima Centauri b was discovered using the radial velocity method. The orbital period of Proxima Centauri b is 11.19 days, and it orbits the star Proxima Centauri, (which has a mass of 0.122 Solar masses, a radius of 0.154 Solar radii, and an effective temperature of 3042 Kelvin).

Problem 18. Which of the following is closest to the semi-major axis of Proxima Centauri b?

Solution: **B: 0.05 AU**

$a(AU)^3/T(yr)^2 = M(Sun) \rightarrow a = (T(yr)^2 \cdot M(Sun))^{1/3}$. Plugging in, we get $a = 0.049AU$, closest to 0.05 AU.

□

Problem 19. Which of the following is closest to the temperature at the surface of Proxima Centauri b? Assume that the surface has an albedo of 0.3, and that the incident radiation is perfectly redistributed around the planet.

Solution: **B: 235K**

$T_{eq} = (L_{star} \cdot (1 - A)/(16 \cdot \sigma \cdot \pi \cdot a^2))^{1/4}$, where $L_star = 4 \cdot \pi \cdot R^2 \cdot \sigma \cdot T^4$. Plugging in (with A = 0.3), we get $T_eq = 238.5K$, closest to 235 K.

□

Problem 20. A star cluster with a main-sequence turn-off at around 6000 K effective temperature is about:

Solution: **D: 10 billion years old**

6000 K is about the temperature of the sun, so if the turn-off point is there then the cluster should be about as old as the total lifetime of the sun.

□

Problem 21. The 21 cm absorption line is a tracer for what kind of gas?

Solution: **A: Cold neutral atomic hydrogen**

21 cm radiation is associated with neutral hydrogen, eliminating all but A and C. The question asks about absorption, so C is wrong because warm neutral gas will emit 21 cm radiation, leaving A.

□

Problem 22. From the shortest distances to the longest, what is the correct order of distance determination techniques?

Solution: **B: Parallax, cepheids, type Ia supernovae**

Because parallax involves resolving physical displacements in the sky, it is limited by the physical resolution of telescopes, so it is best suited for short-enough distances where the transverse displacement is detectable (the closer, the more so). Using Cepheids involves being able to measure the magnitudes of the Cepheids over a period of time and being able to detect the magnitude of the changes in order to determine said period. However, because the absolute magnitude of a Cepheid must be determined by the period, it cannot be used once the oscillations in apparent magnitude become so small that the period cannot be determined. At that point, the Type Ia supernovae become useful because their absolute magnitudes are fixed (given that they occur at a fixed white dwarf mass), so as long as one can see it and determine the apparent magnitude, one has all the info needed to determine distance.

□

The information below applies to the following two questions:

In a particular compact binary system consisting of a black hole and a main sequence star, the black hole has a mass of $23.2 M_{Sun}$ and the main sequence star has mass of $15.6 M_{Sun}$. The two stars are separated by 1 AU.

Problem 23. A clump of gas of mass $1 M_{Sun}$ detaches from the main sequence star. When the gas is 1 km from the black hole, what is its total energy? Neglect viscous forces.

Solution: **A: -3.06E48 J**

We assume the gas follows a closed orbit. Since it is so close to the black hole, we can neglect the gravitational potential energy due to the secondary star. By the virial theorem, $E = -GMm/(2a)$

□

Problem 24. This problem was cancelled.

Problem 25. The $H\alpha$ line ($\lambda_0 = 656.28nm$) of a galaxy is observed to be redshifted to a value $\lambda = 814.35$. Assuming only cosmological redshift, find the approximate distance to the galaxy in Mpc.

Solution: **D: 910Mpc**

Apply the relativistic form of redshift $\frac{\nu}{c} = \frac{(1+z)^2-1}{(1+z)^2+1}$ and apply Hubble's law, $\nu = H_0 \cdot d$.

□

Problem 26. At what redshift would the (average) temperature in the universe have been hot enough to emit 1 nm photons? The current CMB temperature of the universe is 2.73 K.

Solution: **C:** 10^6

One can either use Wien's law or approximate $h\nu \approx kT$. Either way, an answer on the order of 10^6 is reasonable.

□

Problem 27. In April, the Event Horizon Telescope released the first image of the supermassive black hole of M87. The black hole has a diameter of approximately 270 AU and is located at a distance of 16.4 Mpc. At the observed wavelength of 1.3mm, what is the approximate minimum baseline, or effective diameter, required to image the black hole?

Solution: **B:** $2 \cdot 10^4$ **km**
BH apparent size in arcsec is $\frac{270}{16.4} \cdot 10^6$. $\Theta = 1.22\frac{\lambda}{D}$, rearrange and solve for D after converting theta to rad.

□

Problem 28. At optical wavelengths, star formation is strongly obscured by dust. Studies of protoplanetary disks therefore usually observe at infrared or sub-milimeter wavelengths. Why are these observations less strongly affected by interstellar dust?

Solution: **C: Dust grains scatter shorter wavelengths more efficiently than longer wavelengths**

Dust scattering cross sections scale as λ^{-4}, so the answer is C. A is nonsense, B and D assume emission/absorption processes are responsible for the obscuration rather than scattering.

□

Problem 29. A very curious astronomer decided to cover the left half of the objective lens of a telescope with an opaque material. If he points this telescope to a region of the night sky, how will the image generated by this telescope be different from the original image generated by uncovered lenses?

Solution: **D: The astronomer will see almost the same image, but the stars will look fainter**

Since astronomical objects are extremely far from the astronomer, the light rays coming from these objects are parallel when they reach the telescope. Therefore, half of the lens and the entire lens will be able to get the light from the same astronomical objects. However, since the area of the lens is reduced, it will collect less light, so the stars will look fainter. Since the stars are fainter, the astronomer might also not be able to observe some of the stars anymore.

□

Problem 30. This problem was canceled.